Recycle in a Sugarcane Diffuser

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Extraction of sugar from sugar cane

The following is a break down of the process of extracting sugar from shredded cane in a diffuser:

- Chopping sugarcane into fine fibres
- Extract the sugar from the cane fibres. This is done by the following process in the diffuser:
  - Piling the shredded cane into the moving cane bed
  - Releasing water at the top of the moving cane bed
  - Water percolates through the cane, absorbing the sugar from the sugarcane
  - The water that has filtered through the cane is collected in the trays
  - The water in the trays is then pumped up and released into the cane again, this process is repeated many times
- The final process involves evaporating the water in the trays so that only sugar crystals remain

Typical values

- Cane
  - 15% Fibre
  - 15% Sugar
  - 70% Water
- Bed velocity
  - $\approx$ 1m/min
- Percolation rate
  - $\approx$ 0.1 m/min in diffuser
- Bed height
  - 1.5 - 2 m
- Stage length
  - 4.5 - 6 m

Factors influencing recycle

- Controllable factors
  - Bed velocity
  - Bed height
  - Position of sprays
- Non-controllable factors
  - Permeability
  - Length of stage
  - Imbibition

Problem description

We want to investigate the following questions:

- What is optimum recycle fraction that should be used as a target for setting and controlling a diffuser?
- Can a relationship between the controllable variables and non-controllable variables be derived that will enable the factory to achieve the optimum recycle?

1-D model

In order to tackle the problem, we will first consider a 1-D version of a similar problem to help us understand the workings in our problem better.

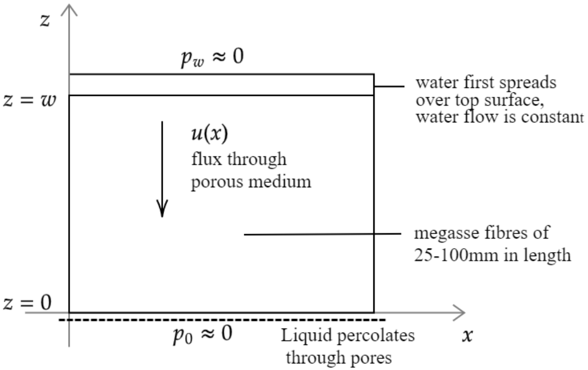


Figure: 1-dimensional model for the flow of fluid through a porous medium

1-D model

The key aspect that we need to consider: How does the liquid flow in the megasse (a porous medium)?

- We will use Darcy's law

Here we will ask the following questions:

- How the liquid flows
  - We will assume that it flows under gravity
- What affects the flow?
  - Permeability of megasse and viscosity of fluid
  - Higher permeability and lower viscosity means increased flow

Assumptions

The following assumptions are made in the 1-D model that we are considering:

- The flow of water through the megasse is only dependent on the z-coordinate.
- The megasse is always saturated.
- The permeability of the megasse is greatest at the surface.
- The atmospheric pressure at the boundaries is approximately zero.

Mathematical Model

Darcy's law: Darcy's law is an empirically formulated equation that describes fluid flow through a porous medium:

$$\underline{u} = -\frac{k}{\mu}(\nabla p - \underline{f})$$

where:
$\underline{u}$ - fluid flux (flow rate per unit area)
k - permeability of the porous medium
μ - dynamic viscosity
p - pressure
$\underline{f}$- external body force per unit volume

We now formulate our equation for 1-D flow. Darcy's law becomes:

$$u(z) = -\frac{k(z)}{\mu}\left(\frac{dp}{dz} + \rho g\right)$$

The permeability k depends on z. The dynamic viscosity μ is constant. Using the continuity equation and noting the relationship between the porosity ϕ, fluid flux u and velocity v, that is:

$$\phi v = u$$

we see that:

$$\frac{du}{dz} = 0$$

We can then formulate the equation for the pressure:

$$k(z)\frac{d^2p}{dz^2} + \frac{dk}{dz}\frac{dp}{dz} + \frac{dk}{dz}\rho g = 0 \quad (1)$$

$$p(0) = 0, p(w) = 0 \quad (2)$$

After solving equation (1) subject to its boundary conditions, we obtain the following dimensionless equation:

$$p(z) = \frac{\int_0^z \left(\frac{1}{k(z')}\right) dz'}{\int_0^1 \left(\frac{1}{k(z')}\right) dz'} - z \tag{3}$$

So given the permeability $k(z)$ we can find the pressure $p(z)$.

Extension to 2-D model

- In the 1-D case the water spreads over the top of the surface before it percolates through the cane
- In our case the bed of sugarcane is moving while the water source remains stationary
- In our model the saturation is not uniform over the x-values -> pressure is dependent on both x and z
- We take into account that the horizontal velocity of the bed ($\approx$ 1m/min) is much larger than the vertical percolation velocity of the fluid ($\approx$ 0.1m/min)
- Our model has several sources of water spaced an equal distance apart

2-D model

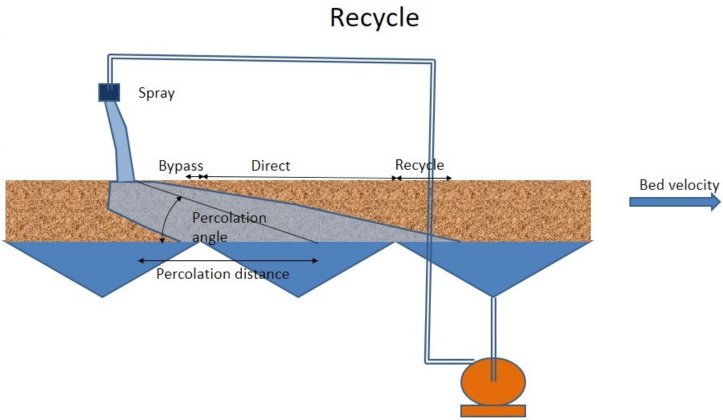


Figure: Recycling in a sugar cane diffuser example presented by R. Loubser

2-D model

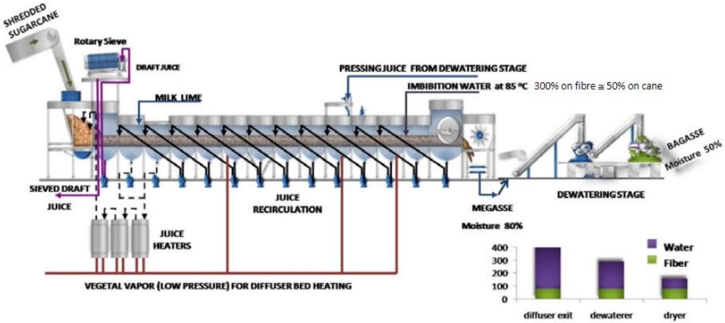
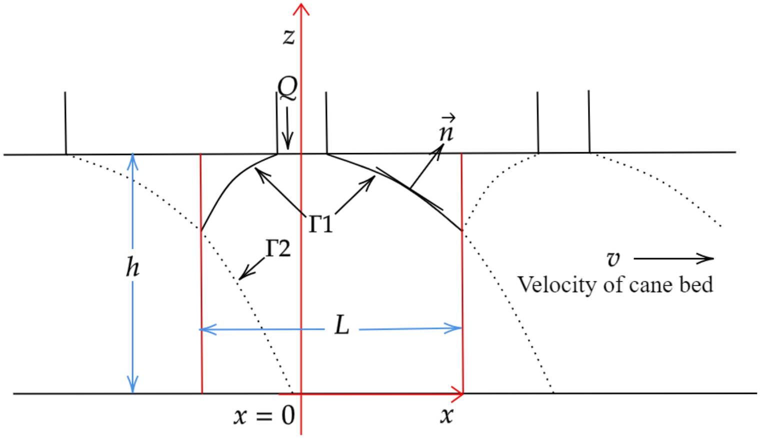


Figure: Recycling in a sugar cane diffuser example presented by R. Loubser

2-D model

Figure: Free boundary value problem



Boundary conditions of 2-D model

For our 2-D model we have our equation from Darcy's Law including the movement of the cane bed and the effect of gravity:

$$\underline{u} = v\underline{i} - \frac{k}{\mu}\nabla(p + \rho g z) \tag{4}$$

We also have the following periodic boundary conditions:

$$p\left(-\frac{L}{2}, z\right) = p\left(\frac{L}{2}, z\right)$$

$$\frac{\partial p}{\partial x}\left(-\frac{L}{2}, z\right) = \frac{\partial p}{\partial x}\left(\frac{L}{2}, z\right)$$

Mathematical Model

As with our 1-D model, we use the continuity equation:

$$\nabla \cdot \underline{u} = 0$$

which, together with equation (4) gives:

$$\frac{\partial^2 p}{\partial x^2} + \frac{1}{k}\frac{dk}{dz}\frac{\partial p}{\partial z} + \frac{\partial^2 p}{\partial z^2} + \frac{1}{k}\frac{dk}{dz}\rho g = 0 \tag{5}$$

where we are working with the permeability k as a function of z.

Non-dimensionalization

We apply the following non-dimensionalization to equation (5):

$$\bar{z} = \frac{z}{h}$$

$$\bar{x} = \frac{x}{L}$$

$$\bar{p} = \frac{p}{\rho g h}$$

$$\bar{k} = \frac{k}{k(w)}$$

Non-dimensionalization

$$\bar{u}_z = \frac{u_z}{a}$$

$$\bar{u}_x = \frac{u_x}{b}$$

Where we choose a to be $a = k\rho g/\mu$ and find $b = aL/h$. We also construct the quantity β in order to draw a relationship between the ratio of the horizontal velocity to the vertical percolation velocity and the ratio of the length to the height:

$$v \div \frac{k\rho g}{\mu} = \frac{L}{h} \times \beta \tag{6}$$

Non-dimensionalization

We now have the following non-dimensionalized equation:

$$\frac{h^2}{L^2}\frac{\partial^2 \bar{p}}{\partial \bar{x}^2} + \frac{1}{\bar{k}}\frac{d\bar{k}}{d\bar{z}}\frac{\partial \bar{p}}{\partial \bar{z}} + \frac{\partial^2 \bar{p}}{\partial \bar{z}^2} + \frac{1}{\bar{k}}\frac{d\bar{k}}{d\bar{z}}\rho g = 0 \tag{7}$$

We notice that the last three terms resemble equation (1) that we solved in the 1-D case (differs by a factor of $\bar{k}$):

$$\frac{h^2}{L^2}\frac{\partial^2 \bar{p}}{\partial \bar{x}^2} + \underbrace{\frac{1}{\bar{k}}\frac{d\bar{k}}{d\bar{z}}\frac{\partial \bar{p}}{\partial \bar{z}} + \frac{\partial^2 \bar{p}}{\partial \bar{z}^2} + \frac{1}{\bar{k}}\frac{d\bar{k}}{d\bar{z}}\rho g}_{\text{1D equation}} = 0$$

Assumptions

We make the following assumptions:

- k (the permeability) is constant
- h^2/L^2 is negligible (very small)

By considering these assumptions equation (7) simplifies to:

$$\frac{\partial^2 \bar{p}}{\partial \bar{z}^2} = 0 \tag{8}$$

since

$$\frac{h^2}{L^2} \ll 1$$

$$\frac{d\bar{k}}{d\bar{z}} = 0$$

On solving equation (8) we get:

$$\bar{p}(\bar{x}, \bar{z}) = A(\bar{x})\bar{z} + B(\bar{x}) \tag{9}$$

2-D model

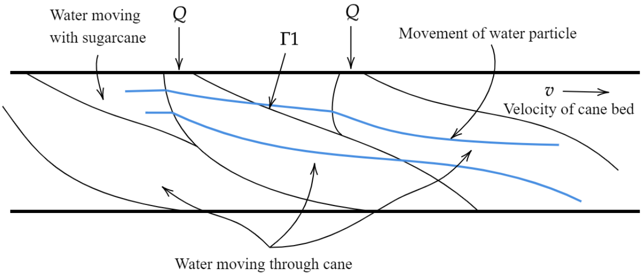


Figure: Free boundary value problem

Non-dimensionalizing the boundary conditions

Conditions on the Γ_1 boundary before non-dimensionalization:

$$p = \rho g(h - z)$$

$$(\underline{u} - v\underline{i}) \cdot \underline{n} = 0$$

We will assume that the Γ_1 boundary can be described by $z = f(x)$. First we non-dimensionalize the condition $p = \rho g(h - z)$:

$$p = \rho g(h - z) \rightarrow \bar{p} = (1 - \bar{z})$$

on the Γ_1 boundary, which means:

$$\bar{p}(\bar{f}(\bar{x})) = 1 - \bar{f}(\bar{x}) \tag{10}$$

If we use this, together with equation (9), we find that:

$$\bar{p}(\bar{f}(\bar{x})) = A(\bar{x})\bar{f}(\bar{x}) + B(\bar{x}) = 1 - \bar{f}(\bar{x}) \tag{11}$$

Non-dimensionalizing the boundary conditions

To non-dimensionalize the condition $(\underline{u} - v\underline{i}) \cdot \underline{n} = 0$ we will use the fact that the Γ_1 boundary can be described by $z = f(x)$.

This implies that any vector normal to that boundary will be parallel to $\underline{n}$, where:

$$\underline{n} = \begin{pmatrix} 1 \\ -\dfrac{1}{f'(x)} \end{pmatrix}$$

Non-dimensionalizing the boundary conditions

From equation (4) we can write $\underline{u}$ in vector form:

$$\underline{u} = \begin{pmatrix} v - \dfrac{k}{\mu}\dfrac{\partial p}{\partial x} \\ -\dfrac{k}{\mu}\dfrac{\partial p}{\partial z} - \dfrac{k}{\mu}\rho g \end{pmatrix}$$

Thus $(\underline{u} - v\underline{i}) \cdot \underline{n}$ becomes:

$$-\frac{k}{\mu}\frac{\partial p}{\partial x} + \frac{k}{\mu}\frac{1}{f'(x)}\left(\frac{\partial p}{\partial z} + \rho g\right) = 0 \qquad (12)$$

Non-dimensionalizing the boundary conditions

We use the same non-dimensionalization for z, x and p as previously, together with

$$\bar{f} = \frac{f}{h}$$

and find that:

$$-\frac{h^2}{L^2}\frac{\partial \bar{p}}{\partial \bar{x}} + \frac{1}{\bar{f}'(\bar{x})}\left(\frac{\partial \bar{p}}{\partial \bar{z}} + 1\right) = 0 \tag{13}$$

on the $\bar{z} = \bar{f}(\bar{x})$ boundary.

Result

On assumption that $\frac{h^2}{L^2} \ll 1$, equation (13) reduces to:

$$\frac{\partial \bar{p}}{\partial \bar{z}} = -1$$

From equation (9) we get that:

$$\frac{\partial \bar{p}}{\partial \bar{z}} = A(\bar{x})$$

Therefore:

$$A(\bar{x}) = -1$$

Furthermore, by using equation (11) we see:

$$B(\bar{x}) = 1$$

Result

Then equation (9) simplifies to:

$$\bar{p}(\bar{z}) = -\bar{z} + 1$$

We note that the solution doesn't satisfy $p = 0$ at $z = 0$. We also find that $\bar{u}_x = \beta$ and that the Darcy flux is $\bar{u}_z = 0$.

Future work

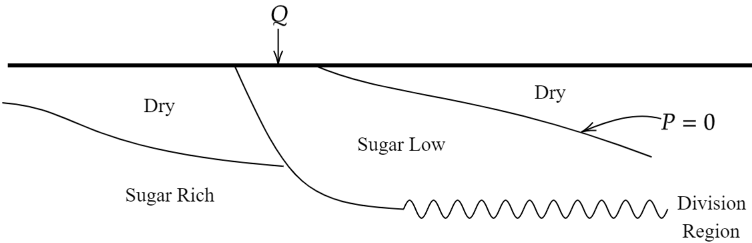


Figure: Free boundary value problem

Conclusion

- Spreading in horizontal direction accelerated by lateral velocity.
- We considered two scenarios:
  - Dry patches
  - Stagnant patches
- We require a mathematical model close to the spray. Long,thin approximation is not valid here.
- Only once we understand the flow behaviour, will we be able to tackle the recycling problem.

The End